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INVESTIGATION OF VARIABLE THERMOPHYSICAL PROPERTY PROBLEM CONCERNING POOL FILM BOILING FROM VERTICAL PLATE WITH PRESCRIBED UNIFORM TEMPERATURE

KANEYASU NISHIKAWA,* TAKEHIRO ITO† and KENICHI MATSUMOTO‡

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Abstract—By the development of laminar two-phase boundary-layer analysis on pool film boiling heat transfer, system parameters and their fundamental effect on heat transfer have been disclosed. However theoretically obtained characteristics of heat transfer do not fairly compare with the experimental data, because the relevant thermophysical properties vary significantly across the vapor film, and the nature of flow process is often turbulent and of non-boundary layer.

In this investigation an analysis was made on pool film boiling heat transfer from vertical plate of uniform temperature to saturated or subcooled liquid, taking into account temperature variation of all properties, and the significance of variable properties in pool film boiling was examined. By getting solutions for water at the pressure of 1, 50, 100 and 200 [bar], effect of variable properties on heat transfer and distribution of velocity and temperature were clarified. The heat-transfer results were compared with the available data of pool film boiling from horizontal cylinder.

NOMENCLATURE

[M] = mass, [L] = length, [T] = time, [θ] = temperature, [F] = [MLT^{-2}] = force, [Q] = [ML^2T^{-2}] = energy;

A,	dimensionless value, equation (40);
B,	dimensionless value, equation (41);
B_v,	constant, equation (24) [$L^{5/4}T^{-1}$];
B_L,	constant, equation (25) [$L^{5/4}T^{-1}$];
c_p,	specific heat at constant pressure [$QM^{-1}\theta^{-1}$];
c_v,	constant, equation (22) [$L^{-3/4}$];
c_L,	constant, equation (23) [$L^{-3/4}$];
C,	dimensionless value, equation (42);
D,	dimensionless value, equation (43);
f,	dimensionless velocity function, equations (20) and (21);
g,	acceleration due to gravity [LT^{-2}];
Gr,	Grashof number, equation (57);
K,	dimensionless value, equation (47);
l,	latent heat of vaporization [Q/M];
Nu,	local Nusselt number, equation (55);
$\overline{Nu}$,	average Nusselt number, equation (54);
$\overline{Nu}_D$,	average Nusselt number for cylinder, equation (59);
Pr,	Prandtl number;
R,	dimensionless value, equation (46);
Sc,	dimensionless subcooling, equation (45);
S_p,	dimensionless superheating, equation (44);
T,	temperature [θ];
ΔT_v,	temperature difference between heating surface and saturated liquid [θ];
ΔT_L,	temperature difference between saturated liquid and bulk liquid [θ];
u,	velocity of x component [LT^{-1}];
v,	velocity of y component [LT^{-1}];
w,	mass flux density [$ML^{-2}T^{-1}$];
x,	co-ordinate along the heating surface [L];
y,	co-ordinate normal to the heating surface [L].

Greek symbols

α,	heat-transfer coefficient [$QL^{-2}T^{-1}\theta^{-1}$];
δ,	thickness of vapor film [L];
η,	dimensionless similarity variable, equations (18) and (19);
θ,	dimensionless temperature, equations (26) and (27);
λ,	thermal conductivity [$QL^{-1}T^{-1}\theta^{-1}$];
μ,	viscosity [FTL^{-2}];
ν,	kinematic viscosity [L^2T^{-1}];
ρ,	density [ML^{-3}];
ψ,	stream function, equations (20) and (21) [L^2T^{-1}].

Subscripts

con,	properties at the saturation temperature;
f,	properties at the film temperature;
i,	vapor–liquid interface;
L,	liquid;
s,	saturated vapor or liquid;
v,	vapor;
var,	variable properties;
w,	heating surface;
∞,	bulk liquid.

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*Professor.

†Associate Professor, Department of Mechanical Engineering, Faculty of Engineering, Kyushu University, Hakozaki, Fukuoka-Shi, Japan.

‡Assistant Professor, Department of Mechanical Engineering, Kurume Technical College, Kurume, Japan.

1. INTRODUCTION

IN FILM boiling, the heating surface being covered with continuous vapor film, the vapor film shares the principal thermal resistance for heat flow. Because of the poor thermal conductivity of the vapor, the temperature difference between the heating surface and bulk liquid is larger than that in the regime of nucleate boiling. The significant temperature-dependence of thermophysical properties (hereafter the term "property" is used for "thermophysical property" throughout this paper) near the saturation temperature must be taken into account in any correlation or theoretical analysis of film boiling heat transfer. Recently many high power density thermal devices may, although mostly in case of accidents, operate in film boiling mode. For liquefied gases and cryogens any solid surface even at room temperature may offer sufficiently high temperature difference for the occurrence of film boiling. Therefore there is a very urgent need to clarify the mechanisms of film boiling heat transfer.

Theoretical analysis of film boiling heat transfer was first proposed by Bromley [1] in 1950, which was based on Nusselt's theory concerning the laminar film condensation. Afterwards some analyses using the concept of the two-phase boundary layer were reported by Koh [2], Sparrow and Cess [3], and Nishikawa and Ito [4] with some success. However, these theories do not adequately account for experimental data in some cases. One of the reasons for the discrepancy might be due to the disregard of the temperature-dependence of properties of vapor and liquid.

On the other hand, McFadden and Grosh [5] presented an analysis of saturated film boiling, taking account the temperature-dependence only of density and specific heat, and concluded that there was a considerable difference between the analyses of variable properties and that of constant properties, but, because of the assumption of the vanishing tangential velocity of fluid at the vapor–liquid interface and of the disregard of the temperature-dependence of viscosity and thermal conductivity, it is open to question whether they could truly assess the effect of variable properties.

In film boiling, as mentioned above, the temperature difference across the vapor film is inevitably so large that the temperature-dependence of properties in the vapor film must be very severe. This situation may be easily understood if one thinks about the large temperature difference itself and the general steep temperature variation of properties of vapor in the proximity of saturation temperature.

In this paper, all relevant properties—density, specific heat, viscosity and thermal conductivity—were regarded as temperature-dependent, and the film boiling heat transfer to saturated or subcooled liquid was analyzed and it was concluded that the effect of variable properties was really remarkable.

2. PHYSICAL MODEL AND CO-ORDINATES SYSTEM

The physical model and the co-ordinate system for the plane vertical heated plate and heated horizontal cylinder are shown in Fig. 1. The heated plate whose surface is kept at a uniform temperature, T_w, is submerged vertically in a stagnant boiling liquid whose temperature is lower than the saturation temperature, T_s, by the degree of subcooling, ΔT_L. The heat-transfer results to be obtained for the vertical plate will be transformed to those for the cylinder by the relation shown in Section 3.3. Several assumptions are made for the derivation of the fundamental equations of conservation law; (1) the vapor–liquid interface is smooth and held at the saturation temperature, (2) the vapor film surrounding the heated plate and the liquid adjacent to it are assumed to have the nature of the boundary layer, (3) the radiative heat transfer is not taken into account, or it, if significant, may be added to the convective component considered here by some appropriate method.

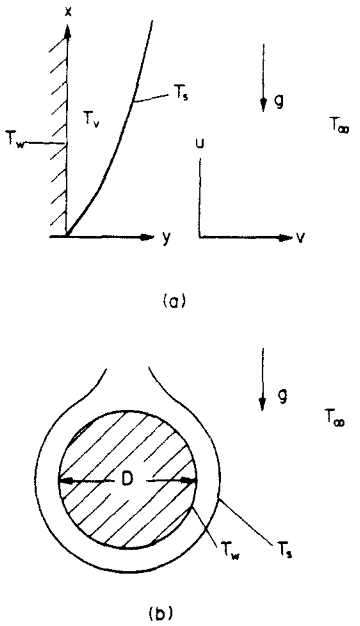

FIG. 1. Physical model and co-ordinate system for (a) plane vertical plate, and (b) horizontal cylinder.

3. ANALYSIS

3.1. *Fundamental equations*

By the physical model and the co-ordinates described above, the fundamental equations of heat transfer for vapor and liquid boundary layer are written as follows.

$$\frac{\partial}{\partial x}(\rho_v u_v)+\frac{\partial}{\partial y}(\rho_v v_v)=0 \tag{1}$$

$$\rho_v u_v \frac{\partial u_v}{\partial x}+\rho_v v_v \frac{\partial u_v}{\partial y}=g(\rho_{L\infty}-\rho_v)+\frac{\partial}{\partial y}\left(\mu_v \frac{\partial u_v}{\partial y}\right) \tag{2}$$

$$\rho_v c_{p_v}\left(u_v \frac{\partial T_v}{\partial x}+v_v \frac{\partial T_v}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda_v \frac{\partial T_v}{\partial y}\right) \tag{3}$$

$$\frac{\partial}{\partial x}(\rho_L u_L)+\frac{\partial}{\partial y}(\rho_L v_L)=0 \tag{4}$$

$$\rho_L u_L \frac{\partial u_L}{\partial x}+\rho_L v_L \frac{\partial u_L}{\partial y}=g(\rho_{L\infty}-\rho_L)+\frac{\partial}{\partial y}\left(\mu_L \frac{\partial u_L}{\partial y}\right) \tag{5}$$

$$\rho_L c_{p_L}\left(u_L \frac{\partial T_L}{\partial x}+v_L \frac{\partial T_L}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda_L \frac{\partial T_L}{\partial y}\right). \tag{6}$$

These must be compatible with the following relations at the vapor–liquid interface.

$$w_i \equiv \rho_v\left(v_v - u_v\frac{\mathrm{d}\delta}{\mathrm{d}x}\right)_i = \rho_L\left(v_L - u_L\frac{\mathrm{d}\delta}{\mathrm{d}x}\right)_i \tag{7}$$

$$(u_v)_i = (u_L)_i \tag{8}$$

$$\left(\mu_v\frac{\partial u_v}{\partial y}\right)_i = \left(\mu_L\frac{\partial u_L}{\partial y}\right)_i \tag{9}$$

$$\left(-\lambda_v\frac{\partial T_v}{\partial y}\right)_i = \left(-\lambda_L\frac{\partial T_L}{\partial y}\right)_i - w_i l \tag{10}$$

$$(T_v)_i = (T_L)_i = T_s. \tag{11}$$

These equations set up the conditions of continuity of mass flux, tangential velocity, surface force, heat flux and thermodynamical equilibrium at the interface. Other conventional boundary conditions can be written as follows.

$$y = 0; \quad u_v = v_v = 0, \quad T_v = T_w \tag{12}$$

$$y \to \infty; \quad u_L \to 0, \quad T_L \to T_\infty. \tag{13}$$

3.2. *Similarity transformation*

At first stream functions ψ_v and ψ_L satisfying the equations (14) through (17) are introduced.

$$u_v = \frac{\rho_{vs}}{\rho_v}\frac{\partial\psi_v}{\partial y}. \tag{14}$$

$$v_v = -\frac{\rho_{vs}}{\rho_v}\frac{\partial\psi_v}{\partial x} \tag{15}$$

$$u_L = \frac{\rho_{Ls}}{\rho_L}\frac{\partial\psi_L}{\partial y} \tag{16}$$

$$v_L = -\frac{\rho_{Ls}}{\rho_L}\frac{\partial\psi_L}{\partial x} \tag{17}$$

Also η is defined as a new independent similarity variable, and it is assumed that the dimensionless velocity function f and dimensionless temperature θ can be written as functions of η only.

$$\eta_v = \frac{c_v}{x^{1/4}}\int_0^y \frac{\rho_v}{\rho_{vs}}\mathrm{d}y \tag{18}$$

$$\eta_L = -\frac{c_L}{x^{1/4}}\int_y^\infty \frac{\rho_L}{\rho_{Ls}}\mathrm{d}y \tag{19}$$

$$\psi_v = B_v x^{3/4} f_v(\eta_v) \tag{20}$$

$$\psi_L = B_L x^{3/4} f_L(\eta_L) \tag{21}$$

$$c_v \equiv \left\{\frac{g\rho_{vs}(\rho_{L\infty} - \rho_{vs})}{4\mu_{vs}^2}\right\}^{1/4} \tag{22}$$

$$c_L \equiv \left\{\frac{g\rho_{Ls}(\rho_{L\infty} - \rho_{Ls})}{4\mu_{Ls}^2}\right\}^{1/4} \tag{23}$$

$$B_v \equiv 4c_v \nu_{vs} \tag{24}$$

$$B_L \equiv 4c_L \nu_{Ls} \tag{25}$$

$$\theta_v(\eta_v) = (T_v - T_s)/\Delta T_v \tag{26}$$

$$\theta_L(\eta_L) = (T_L - T_\infty)/\Delta T_L \tag{27}$$

$$\Delta T_v \equiv T_w - T_s \tag{28}$$

$$\Delta T_L \equiv T_s - T_L. \tag{29}$$

By the above transformation, equations (1) through (6) are reduced to the following system of ordinary differential equations.

$$\left\{\frac{(\rho\mu)_v}{(\rho\mu)_{vs}}\cdot f_v''\right\}' - 2(f_v')^2 + 3f_v f_v'' + \frac{\{(\rho_{L\infty} - \rho)/\rho\}_v}{\{(\rho_{L\infty} - \rho)/\rho\}_{vs}} = 0 \tag{30}$$

$$\left\{\frac{(\rho\lambda)_v}{(\rho\lambda)_{vs}}\cdot\theta_v'\right\}' + 3Pr_{vs}\frac{c_{p_v}}{c_{p_{vs}}}\cdot f_v\theta_v' = 0 \tag{31}$$

$$\left\{\frac{(\rho\mu)_L}{(\rho\mu)_{Ls}}\cdot f_L''\right\}' - 2(f_L')^2 + 3f_L f_L'' + \frac{\{(\rho_{L\infty} - \rho)/\rho\}_L}{\{(\rho_{L\infty} - \rho)/\rho\}_{Ls}} = 0 \tag{32}$$

$$\left\{\frac{(\rho\lambda)_L}{(\rho\lambda)_{Ls}}\cdot\theta_L'\right\}' + 3Pr_{Ls}\frac{c_{p_L}}{c_{p_{Ls}}}\cdot f_L\theta_L' = 0. \tag{33}$$

Also, the equations (7)–(11) of compatibility at the vapor–liquid interface are transformed into the following ones.

$$(f_L)_i = A(f_v)_i \tag{34}$$

$$(f_L')_i = B(f_v')_i \tag{35}$$

$$(f_L'')_i = C(f_v'')_i \tag{36}$$

$$S_p = -3(f_v/\theta_v')_i - D(\theta_2'/\theta_v')_i Sc \tag{37}$$

$$(\theta_v)_i = 0 \tag{38}$$

$$(\theta_L)_i = 1 \tag{39}$$

$$A = KR \tag{40}$$

$$B = K^2 \tag{41}$$

$$C = K^3 R \tag{42}$$

$$D = 1/KR \tag{43}$$

$$S_p \equiv c_{p_{vs}}\Delta T_v / l Pr_{vs} \tag{44}$$

$$S_c \equiv c_{p_{Ls}}\Delta T_L / l Pr_{Ls} \tag{45}$$

$$R \equiv \{(\rho\mu)_{vs}/(\rho\mu)_{Ls}\}^{1/2} \tag{46}$$

$$K \equiv \left[\frac{\{(\rho_{L\infty} - \rho)/\rho\}_{vs}}{\{(\rho_{L\infty} - \rho)/\rho\}_{Ls}}\right]^{1/4}. \tag{47}$$

The transformed boundary conditions can be written as follows.

$$f_v(0) = 0 \tag{48}$$

$$f_v'(0) = 0 \tag{49}$$

$$\theta_v(0) = 1 \tag{50}$$

$$\lim_{\eta_L \to \infty} f_L' = 0 \tag{51}$$

$$\lim_{\eta_L \to \infty} \theta_L = 0. \tag{52}$$

S_p and S_c are dimensionless superheating and subcooling scaled by latent heat of vaporization, R the ratio of fluidity of vapor and liquid and K the ratio of buoyant forces which exist in vapor and liquid. Therefore, the formal system parameters for the present problem are Pr_{vs}, Pr_{Ls}, S_p, S_c, R and K.

3.3. *Heat transfer*

The local and average Nusselt number are found from θ_v as follows.

$$\frac{Nu}{(Gr/4)^{1/4}} = -\frac{(\rho\lambda)_{vw}}{(\rho\lambda)_{vs}}\theta_v'(0) \tag{53}$$

$$\bar{N}u = \tfrac{4}{3} Nu \tag{54}$$

$$Nu \equiv \frac{\alpha x}{\lambda_{vs}} \tag{55}$$

$$\bar{N}u \equiv \frac{\bar{\alpha} x}{\lambda_{vs}} \tag{56}$$

$$Gr \equiv \frac{g \rho_{vs}(\rho_{L\infty} - \rho_{vs}) x^3}{\mu_{vs}^2}. \tag{57}$$

The average Nusselt number for a horizontal cylinder of uniform surface temperature is related approximately to that of a vertical plate of uniform surface temperature by the so-called Hermann's transformation [4].

$$\frac{3}{4} \frac{\bar{N}u}{x^{3/4}} = \frac{1}{1.03} \frac{\bar{N}u_D}{D^{3/4}} \tag{58}$$

$$\bar{N}u_D = \frac{\bar{\alpha}_D D}{\lambda_{vs}}. \tag{59}$$

The relation in terms of heat-transfer coefficient becomes

$$\tfrac{3}{4} \bar{\alpha} x^{1/4} = \frac{1}{1.03} \bar{\alpha}_D D^{1/4}. \tag{60}$$

3.4. *Method of calculation*

The problem is to find the solution for the system of ordinary differential equations (30)–(33) which satisfy the compatibility and boundary conditions (48)–(52) for given six parameters, Pr_{vs}, Pr_{Ls}, S_p, S_c, R and K. Equations were solved numerically and the calculations were performed by FACOM 230-60 digital computer of Kyushu University.

4. RESULTS AND DISCUSSIONS

In order to examine the effect of variable properties systematically when the system pressure, degree of superheating and that of subcooling were varied, solutions were obtained for water at four different pressure levels of 1, 50, 100 and 200 [bar]. Water is one of the substances for which table and chart of properties are fully established for a wide range of pressure and temperature. This is the reason that calculations were performed on water, irrespective of rather rare applications where the film boiling to water is of importance. Properties used in the calculations are shown in Fig. 2 [6] at each level of pressures only for the steam in which the properties vary with temperature more remarkably than those in water. The relevant properties in equations (30)–(33) are density, specific heat, viscosity and thermal conductivity. Some or all of these are regarded as temperature-dependent when solutions with variable property are to be obtained, while in calculation with constant property these were evaluated at the saturation or the film temperature (arithmetic mean between the wall and saturation temperature).

4.1. *Heat-transfer coefficient*

The results of calculation regarding heat-transfer coefficient are summarized in Fig. 3, the abscissa being the degree of superheating the ordinate the heat-transfer coefficient and parameter the degree of subcooling for each level of pressures. In the labels for ordinate, α is the local heat-transfer coefficient for vertical plate, $\bar{\alpha}$ the average heat-transfer coefficient for vertical plate and $\bar{\alpha}_D$ the average heat-transfer coefficient for horizontal cylinder of diameter D, while

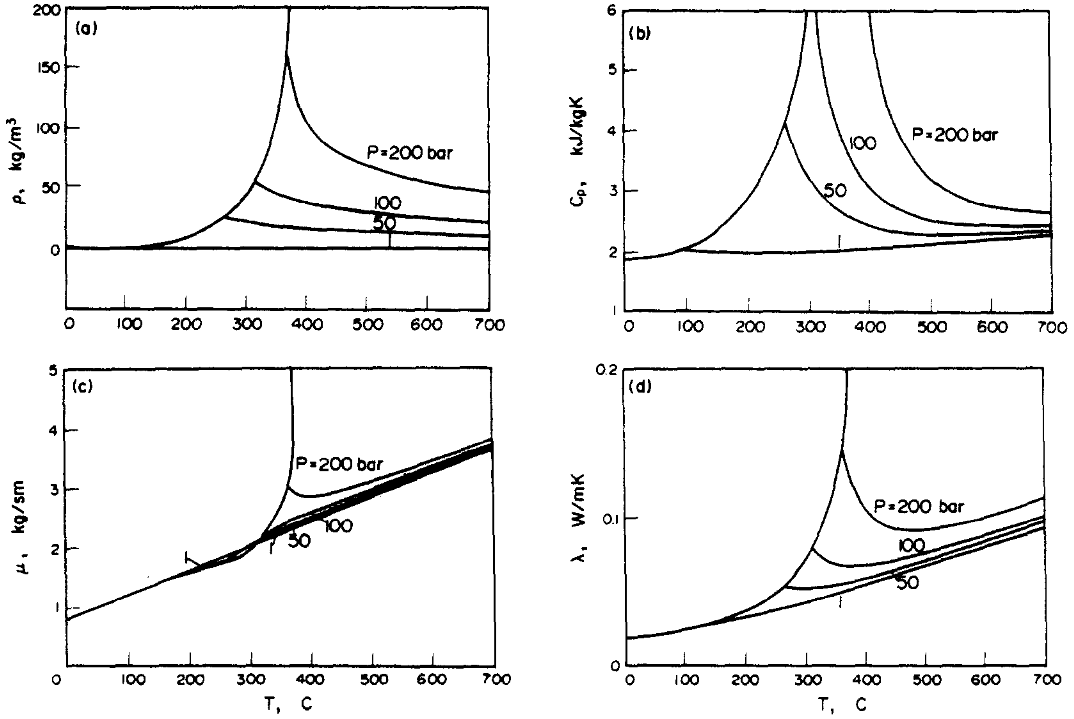

FIG. 2. Properties of steam (a) density, (b) specific heat, (c) viscosity, and (d) thermal conductivity.

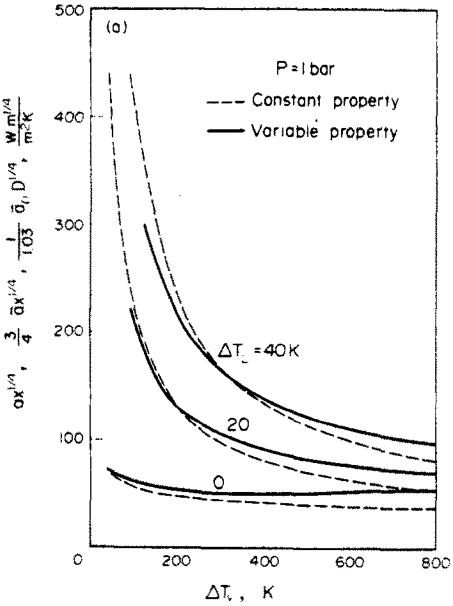

FIG. 3(a). Heat-transfer coefficient ($P = 1$ [bar]).

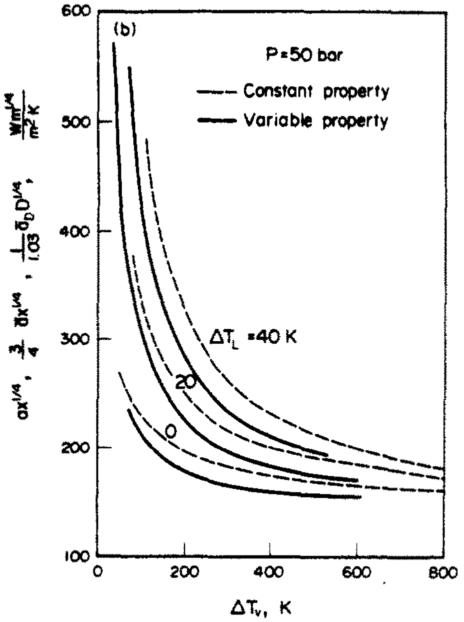

FIG. 3(b). Heat-transfer coefficient ($P = 50$ [bar]).

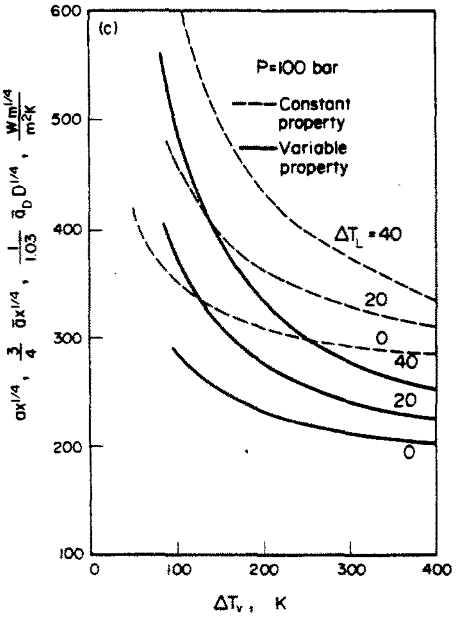

FIG. 3(c). Heat-transfer coefficient ($P = 100$ [bar]).

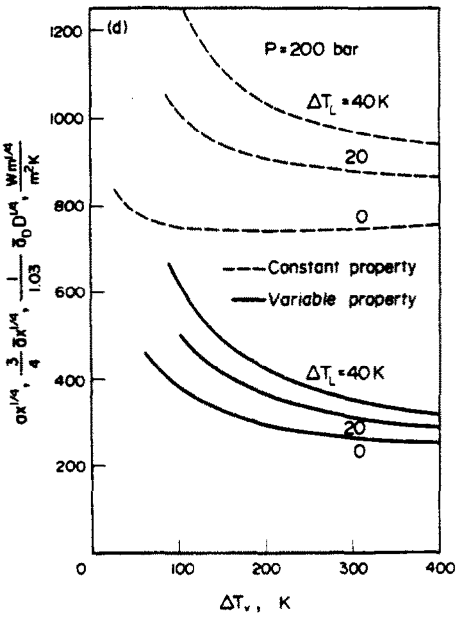

FIG. 3(d). Heat-transfer coefficient ($P = 200$ [bar]).

$\alpha x^{1/4}$, $(3/4)\bar{\alpha}x^{1/4}$ and $\bar{\alpha}_D D^{1/4}/1.03$ have the same value for given macro-parameters, and do not depend on the height x or the diameter D by virtue of the similarity transformation and the Hermann's transformation introduced for cylinder [4]. All the constant property solutions were obtained with properties evaluated at saturation temperature except for Fig. 8 where they were assigned at the film temperature.

At the pressure of 1 [bar] the heat-transfer coefficient for saturated liquid ($\Delta T_L = 0$ [K]) is always larger for variable property treatment than for that of constant property. The difference between the two becomes larger with increase of the degree of superheating. For the degree of subcooling of 20 and 40 [K], up to a certain degree of superheating there exists a region where solution with variable property gives the smaller heat-transfer coefficient. However it remains to be examined whether or not a stable film boiling can exist in such a region of superheating if the minimum heat flux is taken into account. At the pressures of 50, 100 and 200 [bar] the heat-transfer coefficient for variable property is always smaller than one for constant property. As the level of system pressure becomes higher, the difference between these two treatments gets greater, until results for variable property at 200 [bar] do not reach even to 50 [%] of ones for constant property. On the other hand, the heat-transfer coefficient gets larger with the pressure level, if the degree of superheating and subcooling are fixed.

In Fig. 4, the effects of variable properties are

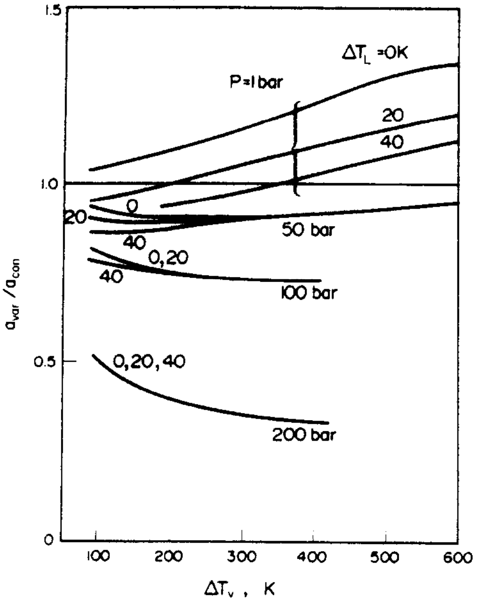

FIG. 4. Solutions with full variable property compared to those with constant property at the saturation temperature.

examined. The ordinate indicates the ratio of the heat-transfer coefficient of variable property, α_{var}, to one of constant property, α_{con}. The figure may give the appearance that the effects of the degree of subcooling become less important as the pressure level increases. But this is due to the way of selection of the ordinate and it is by no means realistic that the effect of degree of subcooling gets weaker at high pressure levels, as is seen clearly in Fig. 3. In such a plot as Fig. 4 one can see the relative importance between the effect of variable property and that of the degree of subcooling. The effect of the degree of subcooling revealed in Fig. 4 coincides with the fact in Fig. 3 that the effect of the degree of subcooling is pronounced at low pressures. As the degree of superheating gets higher, the difference of heat-transfer coefficient between these two treatments should become greater, and the values of the ordinate in Fig. 4 should be widely different from unity. Indeed, as seen in the figure, these expectations are fulfilled except at the pressure of 50 [bar]. However at the pressure of 50 [bar] $\alpha_{var}/\alpha_{con}$ seems to approach unity asymptotically as the degree of superheating increases. This corresponds to the fact in Fig. 3(b) that the distances between two curves swell in the middle range of the degree of superheating. The reason for it, by referring to Fig. 6 explained later, may be that at the pressure of about 50 [bar] the effects of variable property of each properties have a tendency to countervail each other as the degree of superheating increases. At the same time solutions for the pressure of 50 [bar] may be reasonable, considering the fact that $\alpha_{var}/\alpha_{con}$ at the pressure of 1 [bar] is larger than unity and increases with the degree of superheating, while $\alpha_{var}/\alpha_{con}$ at the pressure of 100 and 200 [bar] is smaller than unity and decreases with the degree of superheating. Between 1 and 50 [bar] there must exist a pressure where α_{var} equals α_{con}, that is to say, the effects of variable property of each properties can countervail each other. Of course the pressure itself should depend on the degree of superheating and subcooling.

4.2. *Influence of temperature-dependence of each individual property*

It is difficult to know the influence of temperature-dependence of each property from solutions mentioned above. Therefore calculations were performed under the condition where some particular properties were temperature-dependent, and others constant at some specified temperature.

For this purpose calculations were performed on the following cases where (1) the properties of vapor were variable and those of liquid constant at the saturation temperature, (2) a particular one of properties of vapor was variable and the others constant at the saturation temperature for saturation boiling, (3) viscosity and thermal conductivity were constant at the film temperature $(T_w + T_s)/2$ and others variable for saturation boiling.

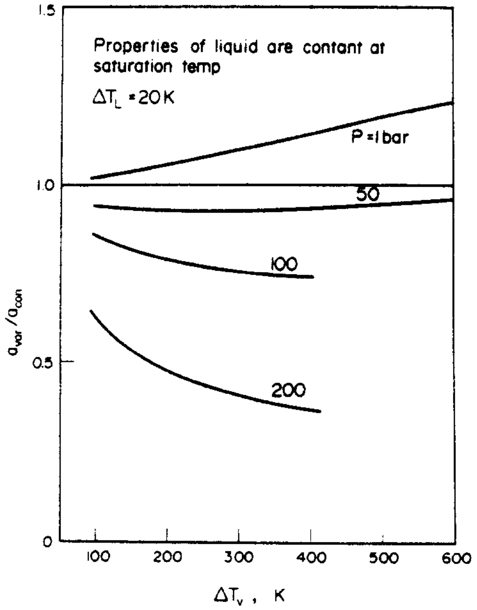

FIG. 5. Solutions with variable property only of vapor compared to those with constant property at the saturation temperature.

Figure 5 presents the results of calculation for the case of (1), the degree of subcooling being 20 [K]. The figure does not indicate any remarkable difference from the results when all properties are variable, i.e. Fig. 4 for $\Delta T_L = 20$ [K]. It may be concluded that the effect of variable property of liquid is less important compared with that of vapor. Results for the case of (2) are shown in Fig. 6. In the figure, the curve 1 corresponds to that in Fig. 4 for $\Delta T_L = 0$ [K]. The curve 2 with variable density is always on the side of $\alpha_{var}/\alpha_{con} < 1$. Therefore the effect of promotion of heat transfer caused by the increase of body force due to the reduced density cannot overcome the effect of deterioration of heat transfer caused by the increase of thickness of vapor film. On the other hand, in the

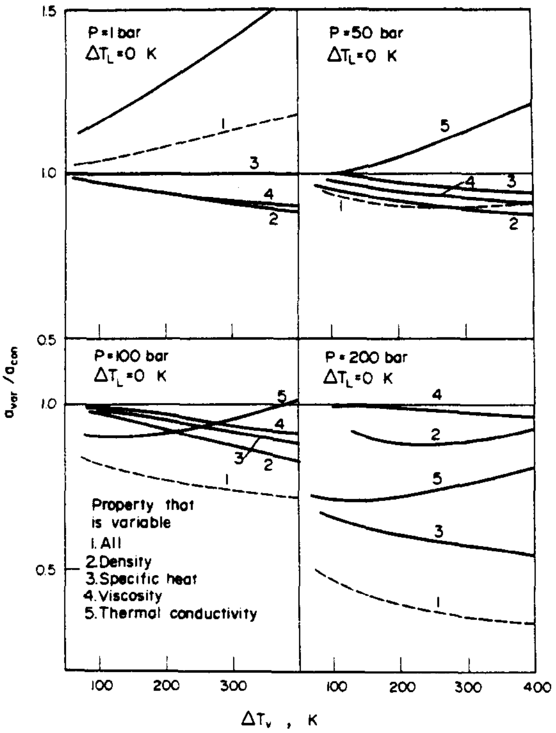

FIG. 6. Individual effect of each properties of vapor.

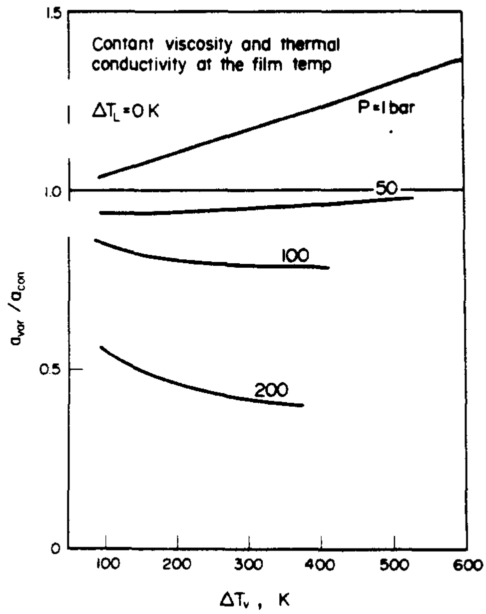

FIG. 7. Solutions with partial variable property (density and specific heat are variable and viscosity and thermal conductivity constant at the film temperature) compared to those with constant property at the saturation temperature.

range of present interest, specific heat is a decreasing function of temperature (negative gradient becomes steeper as pressure increases) in the temperature near saturation for pressures other than 1 [bar], while in the higher temperature region it is slightly increasing function of temperature for low pressure, and assumes nearly horizontal slope for high pressure. Therefore, in curve 3 with variable specific heat α_{var} is always smaller than α_{con} and the tendency gets more emphasized as the pressure increases. Viscosity is an increasing function of temperature except at high pressure and temperature near saturation. Curve 4 with variable viscosity lies in the side of $\alpha_{\text{var}}/\alpha_{\text{con}} < 1$. Thermal conductivity shows almost the same temperature-dependence as specific heat qualitatively and has a strong tendency to be an increasing function of temperature, while only at the temperature near saturation and at the high pressure it has negative slope for temperature. As a result of it, curve 5 expressing the effect of variable thermal conductivity behaves as $\alpha_{\text{var}}/\alpha_{\text{con}} > 1$ at the low pressures, and as $\alpha_{\text{var}}/\alpha_{\text{con}} < 1$ at the high pressures though the effect is lessened as the degree of superheating increases.

For the case of (3), heat-transfer coefficient divided by that of constant property is shown in Fig. 7. The curves are much the same as those in Fig. 4 for $\Delta T_L = 0$ [K], although the former gives $\alpha_{\text{var}}/\alpha_{\text{con}}$ more closer to unity than the latter. This means that the effects of variable viscosity and thermal conductivity, if they are evaluated at the film temperature, are less important than those of variable density and specific heat. By the way, McFadden and Grosh [5] analyzed the film boiling heat transfer from isothermal vertical plate to saturated water and recognized that there is a remarkable difference between solutions with constant property and those with variable property at the pressure near critical. In their analysis only density and specific heat were treated as temperature-dependent, but viscosity and thermal conductivity as constant at film temperature as the case of (3) mentioned above. However an irrational assumption that the tangential velocity u at the vapor–liquid interface is zero were introduced.

4.3. *Comparison of solution with full variable property and that with constant property at the film temperature**

The concept of the film or reference temperature is very useful mean in practice for fluid with moderate variation of relevant properties with temperature. Solution with full variable property was compared to that with constant property at the film temperature in Fig. 8. The former divided by the latter is taken as the ordinate. It is evident that the solution with constant property at the film temperature fails to predict the heat-transfer coefficient at high pressures.

4.4. *Velocity and temperature profile*

Figure 9 shows the velocity and temperature profile in the boundary layer for the pressure of 1 and 200 [bar] and the degree of superheating of about 400 [K]. The abscissa expresses η_v and η_L which are dimensionless y co-ordinate normal to the vertical plate, and the

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*Calculations in this section were performed by the suggestion made by Professor Katto of Tokyo University for which the authors are grateful.

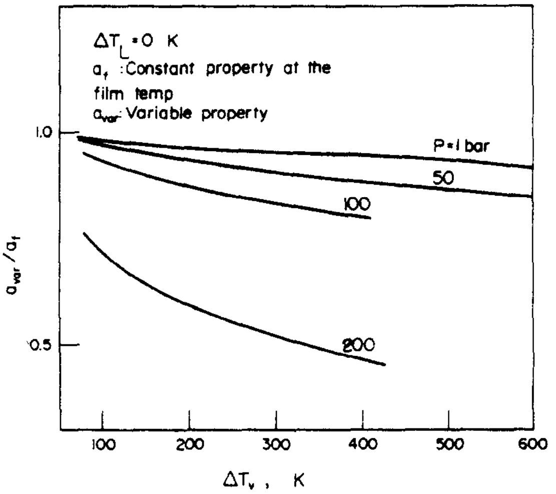

FIG. 8. Solutions with variable property compared to those with constant property at the film temperature.

ordinate the first derivative of the dimensionless velocity function f_v', f_L' and temperature T. f' and η are proportional to tangential velocity u and y co-ordinates respectively with almost the same constants of proportionality for any same pressure, whether or not the properties are temperature-dependent. Irrespective of pressure, the effect of variable property thickens the vapor film and increases the maximum tangential velocity in the vapor film. For the case of variable property at the pressure of 1 [bar], the temperature profile in the vapor film allows the relation of $\partial^2 T/\partial y^2 < 0$. It does not mean a negative right side of the energy equation [equation (3)] and does not represent an exothermic term, but that $(\partial/\partial y)\lambda(\partial T/\partial y)$ is of course positive in conjunction with the variation of thermal conductivity with temperature. At the pressure of 200 [bar] the thickness of temperature boundary layer in liquid for variable property is thinner than for constant property. This appears to contradict the fact that Prandtl number of water is an increasing function of temperature in this range of temperature. The thermophysical parameters contained in the energy equations of liquid are $(\rho\lambda)_L/(\rho\lambda)_{Ls}$, c_{pL}/c_{pLs} and Pr_{Ls}. As Pr_{Ls} is common in both treatments, the other two are thought to have produced such effect. On the other hand the tangential velocity at the vapor–liquid interface seems larger for variable property than for constant property. Therefore the assumption of vanishing tangential velocity at the vapor–liquid interface for variable property treatment is more unreasonable than for constant property treatment.

4.5. *Comparison with available data*

Figure 10 shows comparison between calculated results with full variable property ($P = 1$ [bar]) and authors' experimental data [7, 8] of film boiling from horizontal cylinder to water at the atmospheric pressure. The experimental data plotted in Fig. 10 are for horizontal cylinders of 6 to 16 [mm] in diameter. In the figure, Bromley's experimental data [1] with a cylinder of 8.9 [mm] in diameter at the atmospheric pressure are also plotted. As it is clear from the figure, data of saturation boiling for cylinders with 6 and 8.9 [mm] in diameter are in good agreement with the

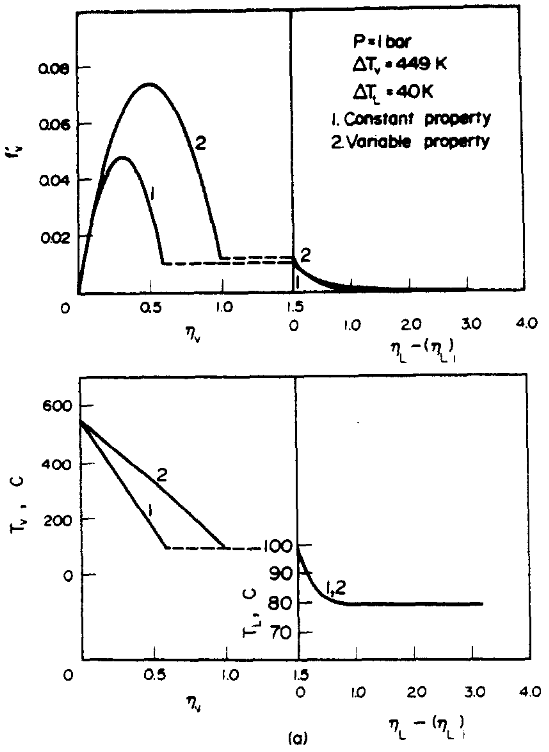

FIG. 9(a). Velocity and temperature profiles ($P = 1$ [bar]).

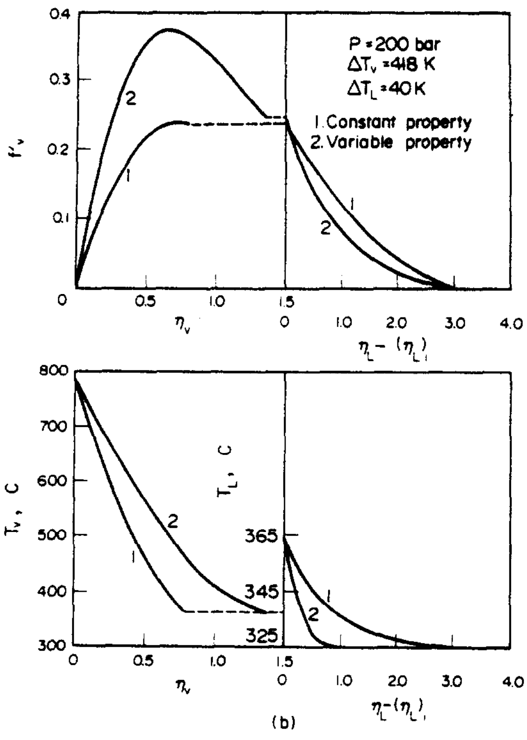

FIG. 9(b). Velocity and temperature profiles ($P = 200$ [bar]).

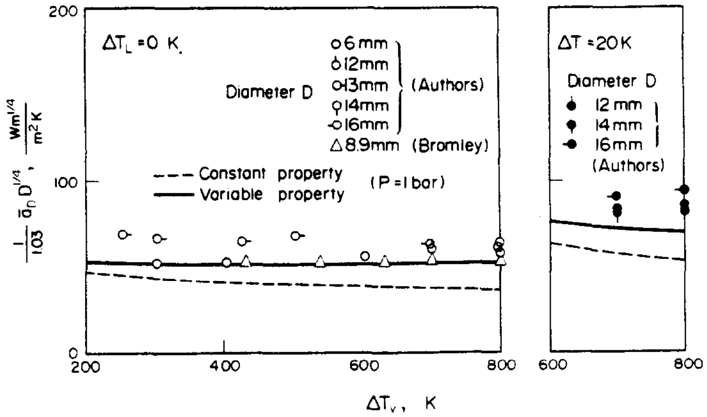

FIG. 10. Comparison between solutions with full variable property and experimental data of horizontal cylinder at atmospheric pressure.

theoretical solutions of variable property. Experimental data with cylinder of diameters other than those mentioned above surpass more or less the theoretical results. The larger the degree of superheating and the diameter of cylinder are, the more the generated vapor is accumulated at the upper portion of the cylinder. Departure of bubbles from there then becomes more violent and disturbance should be produced at the vapor–liquid interface. The increased heat-transfer coefficient might be due to these processes [8]. The discrepancy between the theory and the data in this respect corresponds to the limitation put from the assumption of the laminar boundary-layer treatment. While a good agreement obtained for cylinders of medium size diameter may prove that the proposed analysis of variable property problem is reasonable. Besides, it will be highly necessary to compare the analysis with experimental data at high pressure level where the effect of variable property is significant.

6. CONCLUSION

In film boiling, for high degree of superheating exists inevitably and therefore, the temperature of the vapor film is much higher than saturation temperature. Conventionally what is called the film temperature has been used as the temperature giving the properties. But this is an expediency, for there are no other reasonable measures. Especially when the dependence of the property on temperature is extremely severe, as in the critical region, it will be proper to analyze heat transfer as a problem of variable property.

In this research an analysis of film boiling was made as a variable property problem on the basis of the two-phase boundary-layer theory, and the effect of the variable property was examined in the wide range of the pressure and the degree of superheating and of subcooling for water. Though the effect of variable property manifests various kinds of features according to the value of these parameters, they are well understood from the nature of the variation of the properties with temperature. The calculations in this research were limited to water. The present method of solution must be worked out on many other liquids and the result obtained must be compared to available data to examine the validity of it.

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ETUDE D'UN PROBLEME D'EBULLITION EN FILM D'UN LIQUIDE. A PROPRIETES THERMOPHYSIQUES VARIABLES. SUR UNE PLAQUE VERTICALE A TEMPERATURE CONSTANTE

Résumé—Sur la base d'une analyse du type couche limite laminaire biphasique appliquée au transfert de chaleur par ébullition en film dans un liquide au repos, on a dégagé les paramètres fondamentaux du système et leur influence sur le transfert de chaleur. Cependant les caractéristiques du transfert de chaleur obtenues par voie théorique ne s'accordent pas tout à fait avec les données expérimentales, car les propriétés thermophysiques appropriées varient de façon appréciable à travers le film de vapeur et l'écoulement, de nature souvent turbulente, n'est plus du type couche-limite.

Dans cette étude est analysé le transfert thermique par ébullition en film sur une plaque verticale à température uniforme, dans un liquide au repos, saturé ou sous-refroidi. On a tenu compte de l'influence de la température sur toutes les propriétés thermophysiques et l'importance des propriétés variables dans l'ébullition en film a été examinée. L'obtention de solutions pour l'eau à des pressions de 1, 50, 100 et 200 bars a permis d'éclaircir l'effet des propriétés variables sur le transfert thermique, sur les distributions de vitesse et de température. Les résultats de transfert thermique on été comparés aux données expérimentales disponibles pour l'ébullition en film autour d'un cylindre horizontal.

UNTERSUCHUNG DES EINFLUSSES VERÄNDERLICHER THERMOPHYSIKALISCHER STOFFWERTE AUF DAS BEHÄLTERFILMSIEDEN AN EINER VERTIKALEN PLATTE MIT VORGESCHRIEBENER GLEICHFÖRMIGER TEMPERATUR

Zusammenfassung—Bei der Entwicklung der laminaren Zweiphasen-Grenzschicht-Theorie für den Wärmeübergang beim Behälterfilmsieden konnten Systemparameter und ihr grundsätzlicher Einfluß auf den Wärmeübergang aufgedeckt werden. Das theoretisch ermittelte Wärmeübergangsverhalten stimmt jedoch nicht gut mit den Versuchswerten überein; dies liegt daran, daß wichtige thermophysikalische Stoffwerte sich beträchtlich innerhalb des Dampffilmes ändern und die Strömung häufig turbulent und keine Grenzschichtströmung mehr ist.

Es wurde der Wärmeübergang beim Behälterfilmsieden an einer vertikalen Platte gleichförmiger Temperatur an eine gesättigte oder unterkühlte Flüssigkeit untersucht. Dabei wurde die Temperaturabhängigkeit aller Stoffwerte mit berücksichtigt und untersucht, wie stark sich diese Abhängigkeit auf das Behälterfilmsieden auswirkt. Es wurden Lösungen für Wasser bei Drücken von 1, 50, 100 und 200 bar erhalten, die den Einfluß der veränderlichen Stoffwerte auf den Wärmeübergang, die Geschwindigkeit- und die Temperaturverteilung verdeutlichen. Die Ergebnisse für den Wärmeübergang wurden mit vorhandenen Daten für das Behälterfilmsieden an einem horizontalen Zylinder verglichen.

ИССЛЕДОВАНИЕ ПРОЦЕССА ПЛЕНОЧНОГО КИПЕНИЯ В БОЛЬШОМ ОБЪЕМЕ НА ВЕРТИКАЛЬНОЙ ПЛАСТИНЕ С ЗАДАННОЙ ПОСТОЯННОЙ ТЕМПЕРАТУРОЙ ПРИ ПЕРЕМЕННЫХ ТЕПЛОФИЗИЧЕСКИХ ХАРАКТЕРИСТИКАХ

Аннотация — При использовании аппарата анализа ламинарного двухфазного пограничного слоя для описания теплообмена при пленочном кипении в большом объеме получены параметры системы и выявлено их основное влияние на процесс теплообмена. Однако, полученные теоретические значения характеристик переноса тепла недостаточно хорошо согласуются с экспериментальными данными, т. к. по толщине пленки пара происходит значительное изменение теплофизических свойств, а течение часто носит турбулентный характер, не похожий на течение в пограничном слое. В работе проведен анализ переноса тепла при пленочном кипении жидкости, насыщенной или переохлажденной в большом объеме, на вертикальной пластине постоянной температуры с учетом изменения всех теплофизических характеристик с изменением температуры. Изучалось влияние переменности свойств жидкости на процесс пленочного кипения в большом объеме. На примере воды при давлениях 1, 50, 100 и 200 бар выяснено влияние переменных свойств на перенос тепла и на распространение скорости и темпертауры. Проведено сравнение данных по теплообмену с имеющимися в литературе данными по пленочному кипению в большом объеме на горизонтальном цилиндре.